No class on Thursday

Take-home Final Exam will be available
by 9:15 AM on Thursday

Final Exam is due by 9:30 AM on Tuesday, May 3

6 problems

comprehensive

Root Mean Square value (Effective Value)

$$F_{rms} \stackrel{!}{=} \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt}$$

In this class, $f(t) = A \sin \omega t$

$$f^2(t) = A^2 \sin^2 \omega t$$

$$\omega = 2\pi f = 2\pi \frac{1}{T} \Rightarrow T = \frac{2\pi}{\omega}$$

$$F_{rms} = \sqrt{\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} A^2 \sin^2 \omega t \; dt}$$

$$= \sqrt{\frac{\omega A^2}{2\pi} \underbrace{\boxed{\int_0^{\frac{2\pi}{\omega}} \sin^2 \omega t \; dt}}_{\frac{\pi}{\omega}}}$$

$$\int \sin^2 ax \, dx = \frac{1}{2}x - \frac{1}{4a}\sin 2ax + C$$

$$\int_0^{\frac{2\pi}{\omega}} \sin^2 \omega t \, dt = \left[\frac{1}{2}t - \frac{1}{4\omega}\sin 2\omega t\right]_0^{\frac{2\pi}{\omega}}$$

$$= \frac{1}{2}\frac{2\pi}{\omega} - \frac{1}{4\omega}\sin 4\pi - 0 + 0$$

$$= \frac{\pi}{\omega}$$

$$F_{rms} = \sqrt{\frac{\omega A^2}{2\pi}\frac{\pi}{\omega}} = \sqrt{\frac{A^2}{2}}$$

$$= \frac{A}{\sqrt{2}}$$

For any sinusoidal signal, the rms value is the magnitude divided by $\sqrt{2}$

If $f(t) = 3\cos(\omega t + 27°)$

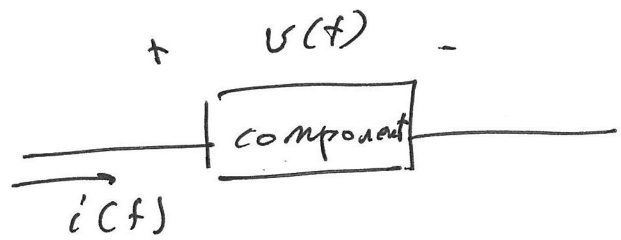
$$F_{rms} = \frac{3}{\sqrt{2}}$$

In practice, we usually specify voltages and currents in terms of their rms (effective) value.

The voltage you have at your home outlet is 115 rms $\Rightarrow$ the actual voltage is $115\sqrt{2}$ max. ≈ 170 peak

$$V_{home} = 170 \sin(120\pi + \theta)\ V$$

Given



where

$$v(t) = V_{max} \sin(\omega t + \theta)$$

$$i(t) = I_{max} \sin(\omega t + \phi)$$

Power absorbed

$$p(t) = v(t) \cdot i(t)$$

$$= V_{max} \sin(\omega t + \theta) \cdot I_{max} \sin(\omega t + \phi)$$

$$= V_{max} I_{max} \sin(\omega t + \theta) \sin(\omega t + \phi)$$

$$\omega = 2\pi f = 2\pi \frac{1}{T} \Rightarrow T = \frac{2\pi}{\omega}$$

$$P_{ave} = \frac{1}{T} \int_0^T p(t)\, dt$$

$$P_{ave} = \frac{1}{T}\int_0^{\frac{2\pi}{\omega}} V_{max} I_{max} \sin(\omega t + \theta)\sin(\omega t + \phi)\,dt$$

$$= \frac{V_{max} I_{max}}{T}\int_0^{\frac{2\pi}{\omega}} \sin(\omega t + \theta)\sin(\omega t + \phi)\,dt$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin(\omega t + \theta)\sin(\omega t + \phi) = \frac{1}{2}\left[\cos(\theta - \phi) - \cos(2\omega t + \theta + \phi)\right]$$

$$P_{ave} = \frac{V_{max} I_{max}}{T}\int_0^{\frac{2\pi}{\omega}} \frac{1}{2}\left[\cos(\theta-\phi) - \cos(2\omega t + \theta + \phi)\right] dt$$

$$= \frac{V_{max} I_{max}}{T}\left\{\underbrace{\frac{1}{2}\int_0^{\frac{2\pi}{\omega}} \cos(\theta-\phi)\,dt}_{\cos(\theta-\phi)\int_0^{\frac{2\pi}{\omega}} dt} - \underbrace{\frac{1}{2}\int_0^{\frac{2\pi}{\omega}} \cos(2\omega t + \theta + \phi)\,dt}_{0}\right\}$$

$$= \frac{V_{max} I_{max}}{T}\,\frac{1}{2}\,\frac{2\pi}{\omega}\cos(\theta-\phi)$$

$$= \frac{V_{max}\; I_{max}}{\cancel{\frac{2\pi}{\omega}}}\,\frac{1}{2}\,\cancel{\frac{2\pi}{\omega}}\cos(\theta-\phi)$$

$$P_{ave} = \frac{V_{max} \; I_{max}}{2} \cos(\theta - \phi)$$

voltage phase (θ) — current phase (φ)

$$= V_{rms} \cdot I_{rms} \underbrace{\cos(\theta - \phi)}_{\text{power factor}}$$

Example

$$v(t) = 100 \sin(\omega t + 30^\circ) \quad V$$

$$i(t) = 40 \sin(\omega t + 60^\circ) \quad A$$

$$P_{ave} = \frac{100}{\sqrt{2}} \frac{40}{\sqrt{2}} \cos(30^\circ - 60^\circ)$$

$$= 2000 \cos(-30^\circ) \quad W$$